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# Viscoplastic analysis of structural polymer composites using stress relaxation and creep data

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**Abstract**

A structural carbon based composite material has been investigated for its high temperature viscoplastic properties using a model based on an overbearing stress concept and using the data obtained from load relaxation and creep. The time dependent viscoplastic properties were obtained at several load and temperature levels. An elastic–viscoplastic constitutive model (proposed by Gates) was used for the modeling efforts. The model is based on an overstress concept appropriate to inelastic properties of composites. The materials parameters for the model are obtained from a set of load relaxation experiments. The model predictions have been compared to the results of creep tests. The results show that the model is capable of predicting the creep behavior at shorter time periods and lower temperatures. As the temperature is increased or as the creep is prolonged the model predictions deviate from the experimental results. 

*Keywords*: Viscoplasticity; A. Polymer matrix composites; B. Creep; B. Stress relaxation

## 1. Introduction

The technological and economic effectiveness of new structural materials depends largely on whether their use meets demands for improved stiffness or strength for a given structural member and ensures its durability under the prescribed loading conditions. The problem of predicting the (time dependant) properties of structural components, the principal among which is reliability, is topical in regard to products made of polymeric composites, since these materials are distinguished by considerable instability and degradation of all mechanical properties (e.g. strength, modulus). When products made of polymer composites are used in real application, they may be subjected, in addition to catastrophic breakdown, to parametric (time dependent) break down (creep) [1]. Prediction of the deformation and long-term strength of polymeric materials has emerged as an independent scientific endeavor based on the modern theories of elastic–viscoplastic behavior [2]. It is well known that the actual mechanical behavior of polymer matrix composite materials (PMCs) is governed by plastic deformation as well as by rheological (time dependent) effects. It can even be said that for many important structural materials rheological effects are more pronounced once the material has been deformed inelastically. The study done by Perzyna [3] underlined the fact that the application of the theory of plasticity, in which rheological effects are disregarded, leads to large discrepancies between the theoretical and experimental results. Hence, there is no need to point out the advantages that can be gained by simultaneous description of plasticity and inelasticity, and the general problem of viscoplasticity. However, the difficulties of combined treatment of plasticity and inelasticity are enormous [3]. As a result of viscous properties of the material the state of the stress and strain is time dependent. The plastic properties on the other hand, make these states depend on the loading path. Thus, the simultaneous introduction of viscous and plastic properties results in dependence on both the load history and time. Hart [4,5] had proposed a set of constitutive laws for inelastic deformation of solids. These laws relate the stress to the deformation history by means of rate dependent state variables. One motivation for the construction of such constitutive laws was to accurately characterize material behavior for use in structural and metal forming processes. Hart's model is primarily a phenomenological approach, and therefore requires extensive experimental input. This need was

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fulfilled by the utilization of the load relaxation test [4]. The internal state variable theory, in general, adapts a couple of internal state variables that embody microstructural effect and requires a kinetic equation which relates the stress to the viscoplastic strain rate [6].

Modeling of composite viscous behavior can benefit from the internal state variable theories. Malvern [7] proposed a time dependent constitutive model to analyze the rate dependent property of graphite/thermoplastic composites. In his model he introduced a viscoplastic component of strain to exist during high strain rate condition. Malvern quantified this viscoplastic component by defining a new parameter, overstress, as the excess of the rate dependent stress over the stress at the same strain in a rate independent test. However, the theoretical prediction of the plastic strain using Malvern's model at low impact velocities was less satisfactory [3]. There was some indication that the simple theory suggested by Malvern needed modification to provide better predictions. The fundamental difference between plasticity and viscoplasticity is that in plasticity there is a unique unloading state associated with each state of homogeneous deformation. This is no longer the case for viscoplasticity, since the unloading behavior depends on the unloading rate. Nishiguchi and Krempl [8,9] extended the theory of viscoplasticity based on overstress as a unified theory that is formulated in terms of strain rates. This extension represents creep, relaxation and rate sensitivity in a unified way. Additive decomposition of the rate of deformation into elastic and inelastic parts was assumed. However, the theory did not include a three dimensional formulation for the yield surface or transient effects associated with loading and unloading.

During the last couple of decades, the stress relaxation tests played a significant role in revealing the material parameters involved in the viscoplastic models. Many researchers such as Lee [5], Hamamy [10], Alexopoulos [11], Grazywinski [12] and Lemaitre [13] addressed this role. Stress relaxation test provides the rate of decrease in stress for any state of stress that results from maintaining a constant strain during uniaxial loading. The data from the load relaxation experiment can produce the strain rate sensitivity data for one state of hardness [4].

Gates [14–16] developed an elastic–viscoplastic model to predict the rate dependent behavior of various types of PMCs. In this model a potential function that accounts for material anisotropy is used to describe the yield surface. This potential function is formed by assuming elastic behavior along the fiber and plane stress condition. Aside from elastic constants five material parameters are required by this model to describe the elastic–viscoplastic behavior. These parameters are found using data from load relaxation tests. Gates model shows a good correlation with short-term creep and other load relaxation tests over a large temperature range.

In this paper, an attempt is made to predict the creep properties of a structural carbon fiber composite material using the model proposed by Gates. Since these composites are primarily used in structural applications they contain a much larger volume fraction of the matrix (resin) material. As a result, they may exhibit a much larger visco-elastic and plastic behavior compared to other aerospace composite materials with higher volume fraction of fibers.

## 2. Mechanical testing

Thornel T-300 12K was used as the carbon fiber material in this study. This fiber is a continuous length, high strength, and high modulus fiber consisting of 12,000 filaments in a one-ply construction. The diameter of each fiber is 7 μm. The fiber surface has been treated to increase the interlaminar shear strength in a resin matrix composite. The fiber tensile strength is 3.65 GPa, and the elastic modulus is 231 GPa.

Thixotropic epoxy was used as the matrix. PTM&W Industries Inc. manufactures this epoxy under the brand name PH2032. It is a medium viscosity, unfilled, light amber laminating resin that is designed for structural production applications. This resin laminates very easily, and wets out fiberglass, carbon, and aramid fibers readily. Used with PH3660 hardener, this system cures at a minimum temperature of 22°C and should be cured at least 24 h before application. The glass transition temperature of the mixture was given by the manufacturer to be 85°C.

Two sheets of (0 90°) pre-form carbon fiber were used to assemble the composite material. The relative weight ratio of the two components of the epoxy was approximately 4:1; the assembled ply-up was left to cure 24 h. Tensile specimens were made according to American Society of Testing and Materials (ASTM) standards [16]. The tabs used to construct the tensile samples were made of a G-10 material.

A 50 kips servo-hydraulic tensile testing machine by mechanical testing system (MTS) and a high temperature furnace (by ATS company) capable of controlling temperature up to 500°C through an OMEGA temperature controller were used. Measurement of strain became possible through a CEA-06-500UW-120 type strain gauge procured by Micro-Measurement Company. The tensile samples were loaded at constant extension rate by a computer program and digital control system "TestStar" of the MTS machine. The data was collected through the data acquisition system within TestStar™. Tensile tests were performed at room temperature (25°C) and two higher temperatures of 40 and 60°C to measure elastic and inelastic (visco-elastic and plastic) properties. The average values of the strength and elastic modulus data are summarized in Table 1. Load relaxation experiments were performed by loading the specimen in tension until a desired load (or total strain) was achieved and then the actuator was forced to an abrupt stop. The relaxation experiment would then be initiated while the extension is kept constant and the load as a function of time is recorded. The load relaxation tests were

Table 1
Mechanical properties at three temperature levels

| $T$ (°C) | $\sigma_u$ (MPa) | $E$ (GPa) |
|---|---|---|
| 20 | 190.3 | 31.02 |
| 40 | 150.6 | 17.93 |
| 60 | 97.2 | 11.032 |

repeated for the three temperatures mentioned earlier and at three different load levels for 25°C and 40°C and at two load levels for 60°C.

## 3. Analytical model

In the following, Gate's model [14] is presented with respect to the research effort described in this paper. Assuming uniaxial loading where the load is not parallel to the fiber direction, the total strain for elastoplastic (time-independent) constitutive relation may be written as a combination of elastic and plastic terms.

$$\varepsilon^{t} = \varepsilon^{e} + \varepsilon^{p} \tag{1}$$

Hooke's law provides the relation between elastic strain and stress, $\varepsilon^{e} = (\sigma/E)$, whereas, the plastic strain, $\varepsilon^{p}$, is expressed by a power law,

$$\varepsilon^{p} = A(\sigma)^{n} \tag{2}$$

where $A$ and $n$ are material constants found from experimental data.

For a rate dependent constitutive relation, viscoplastic strain rate is divided into elastic and viscoplastic components,

$$\dot{\varepsilon}^{t} = \dot{\varepsilon}^{e} + \dot{\varepsilon}^{vp} \tag{3}$$

where elastic strain rate is,

$$\dot{\varepsilon}^{e} = \frac{\dot{\sigma}}{E} \tag{4}$$

while viscoplastic strain rate is decomposed into two terms.

$$\dot{\varepsilon}^{vp} = \dot{\varepsilon}^{p} + \dot{\varepsilon}^{vp'} \tag{5}$$

Differentiating the plastic strain in elastoplastic constitutive relation, Eq. (2) gives the first part of the viscoplastic term, $\dot{\varepsilon}^{p}$.

$$\dot{\varepsilon}^{p} = \begin{cases} An(\sigma)^{n-1}\dot{\sigma} & \text{if} \quad \dot{\sigma} > 0 \\ 0 & \text{if} \quad \dot{\sigma} \le 0 \end{cases} \tag{6}$$

Utilizing the 'overstress' concept provides the second part of the viscoplastic term, $\dot{\varepsilon}^{vp'}$, as,

$$\dot{\varepsilon}^{vp'} = \left[\frac{\langle H \rangle}{K}\right]^{1/m} \tag{7}$$

where $H$ is the overstress, $\langle\ \rangle$ are Macaulay brackets, and $K$ and $m$ are material constants found from experimental data. The overstress, $H = (\sigma - \sigma^{*})$, is considered as a scalar quantity that relates the quasistatic stress, $\sigma^{*}$, to the dynamic or instantaneous stress, $\sigma$, at the same strain level. Thus,

$$\dot{\varepsilon}^{vp'} = \begin{cases} \left[\dfrac{\sigma - \sigma^{*}}{K}\right]^{1/m} & \text{if} \quad \sigma > \sigma^{*} \\ 0 & \text{if} \quad \sigma \le \sigma^{*} \end{cases}. \tag{8}$$

The quasistatic stress is found by using previously defined elastoplastic relation,

$$\varepsilon = \frac{\sigma^{*}}{E} + A(\sigma^{*})^{n} \tag{9}$$

while the dynamic stress is the stress resulting from the time dependent material behavior.

## 4. Material constants

The material constants $K$, $m$, $A$ and $n$ are temperature dependent and are found from experimental data. Here, these constants are determined using load relaxation tests.

During the load relaxation, the quasistatic stress is constant, stress rate is negative and the total viscoplastic strain rate is zero, therefore from Eq. (6) $\dot{\varepsilon}^{p} = 0$, and from Eqs. (3) and (5),

$$\dot{\varepsilon}^{vp} = \dot{\varepsilon}^{vp'} = -\dot{\varepsilon}^{e}. \tag{10}$$

Combining Eqs. (3), (8) and (10) results in Eq. (11).

$$\left[\frac{(\sigma - \sigma^{*})}{K}\right]^{1/m} = -\frac{\dot{\sigma}}{E} = \dot{\varepsilon}^{vp}. \tag{11}$$

Applying polynomial regression to the stress–time data (from a load relaxation test) and differentiating the resulting stress–time curve with respect to time, stress rate, $\dot{\sigma}$, is determined. Applying Gauss–Newton and nonlinear regression methods to Eq. (11) yields $\sigma^{*}$. Once $\sigma$, $\sigma^{*}$ and $\dot{\sigma}$ are known from a plot of the overstress $(\sigma - \sigma^{*})$ against viscoplastic strain rate $(\dot{\varepsilon}^{vp})$ $m$ and $K$ are computed. Note that $m$ and $K$ are independent of the initial applied load of a load relaxation test. Repeating load relaxation tests for different applied loads (strain levels) and calculating the quasistatic stress for each test as mentioned above, a quasistatic stress–strain curve is created. This curve is fit to Eq. (9) to yield the values of $A$ and $n$.

During creep the stress is constant and the stress rate is zero, therefore from Eqs. (3)–(6) and (8) the total strain rate may be written in the form of Eq. (12).

$$\dot{\varepsilon}^{t} = \dot{\varepsilon}^{vp'} = \left[\frac{(\sigma - \sigma^{*})}{K}\right]^{1/m} \tag{12}$$

This relationship is a first order nonlinear differential equation and is coupled to a nonlinear expression of quasistatic stress through Eq. (9). Combined methods of numerical analysis for solving nonlinear equations and differential equations are required to solve this differential equation.

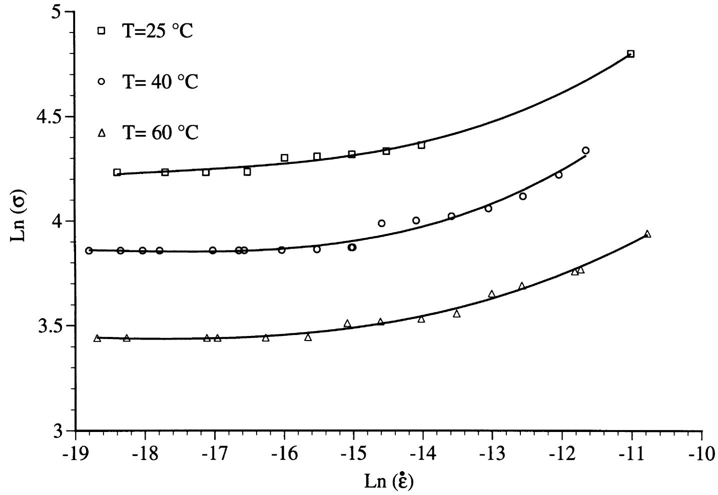


Fig. 1. Logarithm of the stress versus logarithm of the strain rate at three temperatures. Stress is in MPa.

## 5. Results and discussion

The composite used in this study was fabricated without the usage of an autoclave or any automated techniques of composite production; hence, there was no control over the voids concentration, pressure and temperature over the curing period. The objective for the present analysis was processing composites for infrastructure application and as a result the mechanical properties are representative of the non-ideal conditions associated with such processes. To obtain a good statistical distribution for the mechanical testing results, a number of samples (at least four) were tested to failure. Following the testing standards, the results were considered to be repeatable, since the two averages for the properties did not differ by more than two standard deviations [17].

A least squares polynomial regression was used to fit the relaxation data. The derivative of the polynomial was used to compute the viscoplastic strain rate. Fig. 1 shows the logarithm of stress versus logarithm of viscoplastic strain rate for three temperatures at a stress level of approximately 50% of the ultimate stress. Utilizing the values for quasi-static stress, plots of logarithm of overstress against logarithm of viscoplastic strain rate, Fig. 2, and logarithm of quasistatic stress against logarithm of plastic strain, Fig. 3, were generated and approximated as linear equations. Slope and $y$-intercept of these lines yield the values of $m$, $n$, logarithm of $K$ and logarithm of $A$. These values are shown in Table 2.

A study of the model parameters variation, ($K$, $n$, $M$, and $A$), with temperature variation was performed by Gates [16], where he repeated the model simulation for a temperature range of 23–200°C. He also varied the loading condition into tension and compression, finally he repeated these variations for two different composites IM7/5260 and IM7/8320. However, no clear direct trend was found

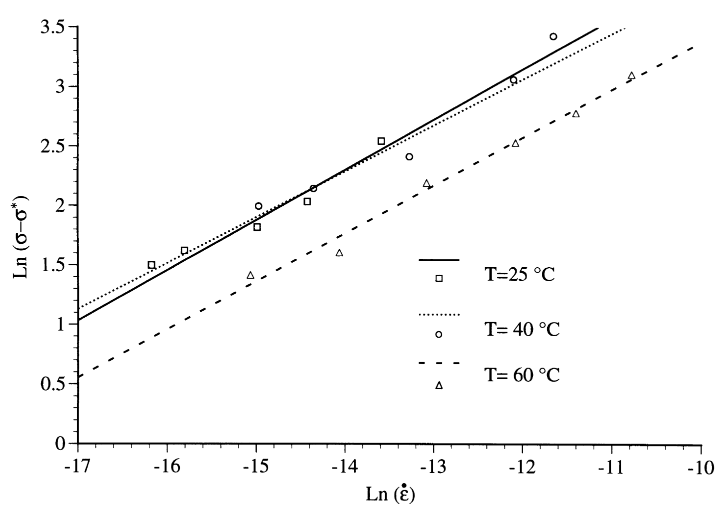


Fig. 2. Logarithm of overstress versus logarithm of strain rate at three temperatures. These curves are used to determine the material constants, $K$ and $m$, at each temperature. Overstress is in MPa.

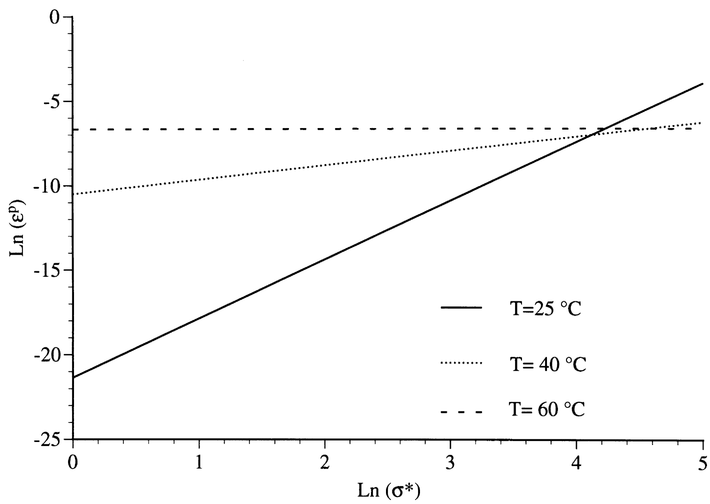


Fig. 3. Logarithm of plastic strain versus logarithm of quasistatic stress at three temperatures. These curves are used to determine the material constants, *A* and *n*, at each temperature. Quasistatic stress is in MPa.

between the temperature variation and the values of the constants. Gates reported one consistent trend that of a remarkable increase in *A* as $T_g^\circ$ of the material was approached. This trend was reported in the current work: when approaching 60° which is closer to the glass transition temperature of the composite, *A* reached a larger value. ($T_g = 85$°C). Gates used a parametric approach to investigate the effect of the temperature by fixing the value of one of the constants throughout the temperature variation, and then investigating the relation between the temperature and other parameters [16]. This ambiguity suggests that there are other factors affecting those constants such as the stress levels, laminates direction, and the glass transition temperatures.

The overstress significantly affects the magnitude of the viscoplastic strain rate as was investigated by Lemaitre [2]. The results of Lamaitre [2], clearly emphasize that the overstress value affects the magnitude of the viscoplastic strain rate in both relaxation and hardening phenomenon, and a higher overstress will result in a higher plastic strain rate.Similar conclusions can also be drawn out of Gates' results. The significance of the overstress in the current work is shown in Fig. 3, which coincides with the previous conclusions of both Lamaitre and Gates.

Fig. 4 presents the comparison of the experimental results and the theoretical model for short-term creep tests for the three temperatures. Each creep test was performed at the initial stress of one half of the ultimate stress of that temperature. The results for the theoretical model were achieved by applying the material constants (found from the load relaxation tests as described above) to Eqs. (9) and (12) and by solving these two equations numerically through the use of fourth-order Runge–Kutta and modified bisection methods. The intent behind showing these figures was to establish confidence in the predictive capabilities of the model. Prediction of short-term creep behavior is a good measure of the model's full capability because the material constants used for the creep prediction were found from the stress relaxation procedures illustrated before. The results show very good agreement between the experiment and the model for the room temperature behavior; however, at higher temperatures (40 and 60°C) there are maximum discrepancies of 15 and 20% between the experiment and the model, respectively. This shows, as the temperature increases the model tends to predict higher strains compared to the actual creep data.

Table 2
Material constants at three temperature levels

| *T* (°C) | *m* | *K* (MPa) | *n* | *A* (MPa)$^{-n}$ |
|---|---|---|---|---|
| 20 | 0.3648 | 1267.8 | 0.8664 | $2.755\times10^{-5}$ |
| 40 | 0.3672 | 2094.4 | 3.502 | $5.306\times10^{-10}$ |
| 60 | 0.3487 | 1330.1 | 0.02674 | $1.270\times10^{-3}$ |

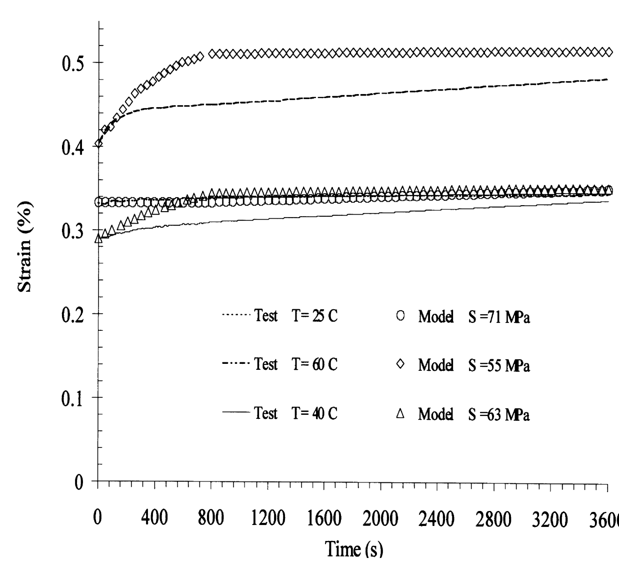


Fig. 4. Comparison of test results and model predictions for creep tests at three temperatures.

## 6. Conclusions

The present work describes the computation of the visco-plastic behavior of PMCs using load relaxation and creep data. Load relaxation tests were performed at different load levels and temperatures. An elastic–viscoplastic constitutive model (proposed by Gates) was used for the modeling efforts. The temperature dependent material constants were obtained by applying the results from load relaxation tests to the theoretical model. Short-term creep tests for different temperatures were performed using strain gauge method. The experimental results for the short-term creep tests were compared to the results of the model. The theoretical creep curves, generated through the material constants found from stress relaxation tests, showed a close agreement with experimental data (especially at room temperature), and thus, present a potential as a design tool for durability prediction of composites for infrastructure industry applications. The stress relaxation based model for generating creep data is significantly more time and cost efficient than traditional tensile creep approaches.

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